### CS 600.226: Data Structures Michael Schatz

Sept 14 2018 Lecture 7. More Complexity



# Agenda

- I. Review HWI
- 2. Introduce HW 2
- 3. Recap & continuation on complexity

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment01/assignment01.md

#### **Assignment 1: Warming Up**

- Out on: September 7, 2016
- Due by: September 14, 2016 before 10:00 pm
- · Collaboration: None
- · Grading:
  - Functionality 65%
  - ADT Solution 30%
  - Solution Design and READMDE 5%
  - Style 0%

#### **Overview**

The first assignment is mostly a warmup exercise to refresh your knowledge of Java and an ADT problem to start you thinking more abstractly about your data.

# GradeScope.com Entry Code: MDJYER



# Agenda

- I. Review HWI
- 2. Introduce HW 2
- 3. Recap & continuation on complexity

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

### **Assignment 2: Arrays of Doom!**

Out on: September 14, 2018 Due by: September 21, 2018 before 10:00 pm Collaboration: None Grading:

Functionality 65% ADT Solution 20% Solution Design and README 5% Style 10%

#### Overview

The second assignment is mostly about arrays, notably our own array specifications and implementations, not just the built-in Java arrays. Of course we also once again snuck a small ADT problem in there...

**Note:** The grading criteria now include **10% for programming style**. Make sure you use <u>Checkstyle</u> with the correct configuration file from <u>Github</u>!

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

#### **Problem 1: Revenge of Unique (30%)**

You wrote a small Java program called Unique for Assignment 1. The program accepted any number of command line arguments (each of which was supposed to be an integer) and printed each unique integer it received back out once, eliminating duplicates in the process.

For this problem, you will implement a new version of Unique called *UniqueRevenge* with two major changes:

- First, you are no longer allowed to use Java arrays (nor any other advanced data structure), but you can use our Array interface and our SimpleArray implementation from lecture (also available on github)
- Second, you're going to modify the program to read the integers from standard input instead of processing the command line.

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

#### Example:

```
$ java UniqueRevenge
192
3
1 4
  5 3
6
 0
<ctrl-d> or <ctrl-z>
1
9
2
3
4
5
6
0
```

#### Hints

- Reading numbers from standard input can be accomplished using a java.util.Scanner object that has been wrapped around System.in which is Java's name for the standard input stream.
- Make sure you hit return one last time at the end of your input and only then signal end-of-file with the appropriate key-combination for your operating system (this restriction doesn't apply when you use I/O redirection to give input to the program, a highly recommended practice for testing).
- You will have to process an unbounded number of inputs, which requires that you keep track of how "full" the array is. When nothing fits into the array anymore, you'll have to "grow" it somehow. The best approach is to double the size of the array when you are out of space. (We'll talk about the reasons for this in lecture next week.)
- Do not try to change everything at once, there are too many "moving parts" to get things right that way. Instead, choose one thing to change,for example just the way input is given to the program, finish that, test it, and only then move on to the next thing. **Remember: Baby steps!**

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

```
import java.util.Scanner;
public class PrintInts {
    public static void main(String[] args) {
        Scanner s = new Scanner(System.in);
        while (s.hasNextInt()) {
            int i = s.nextInt();
            System.out.println("found: " + i);
            }
        }
}
```

<pre>\$ java PrintInts 1 2 3 4 5 found: 1 found: 2 found: 3 found: 4</pre>	<pre>\$ seq 1 5 &gt; nums \$ cat nums 1 2 3 4</pre>	<pre>\$ seq 1 1000 &gt; nums \$ head -2 nums 1 2 \$ tail -2 nums 999</pre>
6 7 8 found: 6 found: 7 found: 8	<pre>found: 1 found: 2 found: 3 found: 4 found: 5</pre>	<pre>\$ head -2 results found: 1 found: 2 \$ tail -2 results found: 999</pre>
found: 9 000 found: 0		found: 1000 <b>\$ wc -1 results</b> 1000 results

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

 First, you are no longer allowed to use Java arrays (nor any other advanced data structure), but you can use our Array interface and our SimpleArray implementation from lecture (also available on github)

Wait a second, Im only allowed to use SimpleArrays, but the constructor requires giving a size.... How do I know how big to make it???

 Call the constructor with an initial buffer size, and then grow the buffer as needed. Make sure to keep track of how many slots are really used.

Wait a second, how do I grow a buffer????

- $\odot$
- Make a new array that is larger, copy everything over.

Wait a second, how much bigger???



Doubling is usually a good rule (wait a few weeks for a formal analysis)

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

#### Problem 2: Flexible Arrays (20%)

Develop an algebraic specification for the abstract data type FlexibleArray which works like the existing Array ADT for the most part **except** that both its **lower** and its **upper** index bound are set when the array is created. The lower as well as upper bound can be **any** integer, provided the lower bound is **less than or equal** the upper bound.

Write up the specification for FlexibleArray in the format we used in lecture and **comment** on the design decisions you had to make. Also, tell us what kind of array **you** prefer and why.

#### Hints

- A FlexibleArray for which the lower bound equals the upper bound has exactly one slot.
- Your FlexibleArray is **not** the Array ADT we did in lecture; it doesn't have to support the exact same set of operations.

# Array ADT

<pre>adt Array uses Any, Integer defines Array<t: any=""></t:></pre>	Uses two related ADTs
<pre>operations new: Integer x T&gt; Array<t> get: Array<t> x Integer&gt; T put: Array<t> x Integer x T&gt; length: Array<t>&gt; Integer</t></t></t></t></pre>	Defines method signatures Array <t></t>
<pre>axioms     get(new(n, t), i) = t     get(put(a, i, t), j) = (if i = j     length(new(n, t)) = n     length(put(a, i, t)) = length(a)</pre>	Enforced by asserts then t else get(a, j))
<pre>preconditions     new(n, t): 0 &lt; n     get(a, i): 0 &lt;= i &lt; length(a)     put(a, i, t): 0 &lt;= i &lt; length(a)</pre>	Enforced by exceptions

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

#### Problem 3: Sparse Arrays (35%)

A **sparse** array is an array in which **relatively few** positions have values that differ from the initial value set when the array was created. For sparse arrays, it is wasteful to store the value of **all** positions explicitly since **most of them never change** and take the default value of the array. Instead, we want to store positions that **have actually been changed**.

For this problem, write a class SparseArray that implements the Array interface we developed in lecture (the same interface you used for Problem 1 above). **Do not modify the Array interface in any way!** Instead of using a plain Java array like we did for SimpleArray, your SparseArray should use a **linked list** of Node objects to store values, similar to the ListArray from lecture (and available in <u>github</u>). However, your nodes no longer store just the **data** at a certain position, they also store **the position itself**!

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

# Here's a rough outline of how your implementation could work:

- Start with an empty list (instead of the complete list we built in the constructor of ListArray).
- For put, check if the relevant position has been modified before (meaning a Node object exists for that position); if not, add a Node to the list for the position and its new value; otherwise update the correct Node to the new value.
- For get, check if the relevant position has been modified before; if not, return the default value; otherwise, return the value found in the relevant Node object.

Important: Your Node class must be nested inside your SparseArray class with private visibility! Clients should not be able to "touch" Node objects in any way! new SparseArray(10, "Mike") SparseArray int length: 10 String default: Mike Node list: null sa.put(5, "Peter") SparseArray int length: 10 String default: Mike Node list: Node int pos: 5 String data: Peter Node next: null

sa.put(8, "Kelly")

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

# Here's a rough outline of how your implementation could work:

- Start with an empty list (instead of the complete list we built in the constructor of ListArray).
- For put, check if the relevant position has been modified before (meaning a Node object exists for that position); if not, add a Node to the list for the position and its new value; otherwise update the correct Node to the new value.
- For get, check if the relevant position has been modified before; if not, return the default value; otherwise, return the value found in the relevant Node object.

Important: Your Node class must be nested inside your SparseArray class with private visibility! Clients should not be able to "touch" Node objects in any way! SparseArray int length: 10 String default: Mike Node list: Node int pos: 8 String data: Kelly Node next: Node int pos: 5

sa.put(8, "Kelly")

sa.put(5, "James")

String data: Peter

Node next: null

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

# Here's a rough outline of how your implementation could work:

- Start with an empty list (instead of the complete list we built in the constructor of ListArray).
- For put, check if the relevant position has been modified before (meaning a Node object exists for that position); if not, add a Node to the list for the position and its new value; otherwise update the correct Node to the new value.
- For get, check if the relevant position has been modified before; if not, return the default value; otherwise, return the value found in the relevant Node object.

Important: Your Node class must be nested inside your SparseArray class with private visibility! Clients should not be able to "touch" Node objects in any way! *SparseArray* int length: 10 String default: Mike Node list: *Node* int pos: 8 String data: Kelly Node next:

sa.put(5, "James")

Node

int pos: 5 String data: James Node next: null

https://github.com/schatzlab/datastructures2018/blob/master/assignments/assignment02/README.md

# Here's a rough outline of how your implementation could work:

- Start with an empty list (instead of the complete list we built in the constructor of ListArray).
- For put, check if the relevant position has been modified before (meaning a Node object exists for that position); if not, add a Node to the list for the position and its new value; otherwise update the correct Node to the new value.
- For get, check if the relevant position has been modified before; if not, return the default value; otherwise, return the value found in the relevant Node object.

Important: Your Node class must be nested inside your SparseArray class with private visibility! Clients should not be able to "touch" Node objects in any way!



sa.get(3) => "Mike"

### Introduction to Checkstyle

#### http://checkstyle.sourceforge.net/

2. bash mschatz@schatzmac:23:11:48:~/Dropbox/Documents/Teaching/2016/JHU/DataStructures/Lectures/02.Practicals \$ java - jar checkstyle-6.15all.jar -c cs226\_checks.xml HelloWorld.java Starting audit... [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:1: Missing a Javad oc comment. [JavadocType] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:1:1: Utility class es should not have a public or default constructor. [HideUtilityClassConstructor] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:2:1: '{' at column 1 should be on the previous line. [LeftCurly] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:3: 'method def mod ifier' have incorrect indentation level 2, expected level should be 4. [Indentation] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:3:3: Missing a Jav adoc comment. [JavadocMethod] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:3:33: 'String' is followed by whitespace. [NoWhitespaceAfter] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:4: 'method def lcu rly' have incorrect indentation level 2, expected level should be 4. [Indentation] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:4:3: '{' at column 3 should be on the previous line. [LeftCurly] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:5: 'method call' c hild have incorrect indentation level 4, expected level should be 8. [Indentation] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:5: 'method def' ch ild have incorrect indentation level 4, expected level should be 8. [Indentation] [ERROR] /Users/mschatz/Dropbox/Documents/teaching/2016/JHU/DataStructures/Lectures/02.Practicals/HelloWorld.java:6: 'method def rcu rly' have incorrect indentation level 2, expected level should be 4. [Indentation] Audit done. Checkstyle ends with 11 errors. mschatz@schatzmac:23:11:52:~/Dropbox/Documents/Teaching/2016/JHU/DataStructures/Lectures/02.Practicals \$

\$ java -jar datastructures2018/resources/checkstyle-8.12-all.jar \
 -c datastructures2018/resources/cs226\_checks.xml HelloWorld.java

### Google's Java Style Guide



This document serves as the **complete** definition of Google's coding standards for source code in the Java<sup>™</sup> Programming Language. A Java source file is described as being *in Google Style* if and only if it adheres to the rules herein.

### cs226\_checks.xml (I)

```
<!-- maximum 2000 lines by default -->
<module name="FileLength"/>
<!-- tabs are not popular in Java -->
<module name="FileTabCharacter"/>
<!-- no trailing whitespace, evil -->
<module name="RegexpSingleline">
   <property name="format" value="\s+$"/>
   <property name="message" value="Line has trailing whitespace."/>
</module>
<module name="TreeWalker">
    <!-- enforce Javadoc but not for private stuff -->
    <module name="JavadocMethod">
       <property name="scope" value="protected"/>
    </module>
    <module name="JavadocType">
       <property name="scope" value="protected"/>
    </module>
    <module name="JavadocVariable">
       <property name="scope" value="protected"/>
    </module>
    <module name="JavadocStyle">
       <property name="scope" value="protected"/>
       <!-- empty tags are not okay -->
       <property name="checkEmptyJavadoc" value="true"/>
    </module>
    <!-- being super-picky here, like Google -->
    <module name="JavadocTagContinuationIndentation"/>
```

**.** 

### cs226\_checks.xml (2)

```
<!-- various naming conventions -->
<module name="ConstantName"/>
<module name="LocalFinalVariableName"/>
<module name="LocalVariableName"/>
<module name="MemberName"/>
<module name="MethodName"/>
<module name="PackageName"/>
<module name="ParameterName"/>
<module name="StaticVariableName"/>
<module name="TypeName"/>
<module name="CatchParameterName"/>
<module name="CatchParameterName"/>
<module name="InterfaceTypeParameterName"/>
<module name="MethodTypeParameterName"/>
```

```
<!-- enforce sane imports -->
<module name="AvoidStarImport"/>
<module name="IllegalImport"/>
<module name="RedundantImport"/>
<module name="UnusedImports"/>
```

```
<!-- size violations -->
```

```
<module name="AnonInnerLength"/> <!-- default 20 lines -->
<module name="LineLength"/> <!-- default 80 chars -->
<module name="MethodLength"/> <!-- default 150 lines -->
<module name="ParameterNumber"/> <!-- default 7 parameters -->
<module name="OuterTypeNumber"/> <!-- default 1 per file -->
```

• • •

### cs226\_checks.xml (3)

```
<!-- whitespace checks -->
<module name="EmptyForInitializerPad"/>
<module name="EmptyForIteratorPad"/>
<module name="EmptyLineSeparator">
    <property name="allowNoEmptyLineBetweenFields" value="true" />
</module>
<module name="GenericWhitespace"/>
<module name="MethodParamPad"/>
<module name="NoLineWrap"/>
<module name="NoWhitespaceAfter"/>
<module name="NoWhitespaceBefore"/>
<module name="OperatorWrap"/>
<module name="ParenPad"/>
<module name="TypecastParenPad"/>
<module name="WhitespaceAfter"/>
<module name="WhitespaceAround">
  <!-- empty methods look better this way -->
  <property name="allowEmptyMethods" value="true" />
  <property name="allowEmptyConstructors" value="true" />
</module>
<!-- sane use of modifiers (sane is a relative term) -->
```

```
<module name="ModifierOrder"/>
<module name="RedundantModifier"/>
```

```
<!-- block checks -->
<module name="AvoidNestedBlocks"/>
<module name="EmptyBlock"/>
<module name="EmptyCatchBlock"/>
<module name="LeftCurly"/>
<module name="NeedBraces"/>
<module name="RightCurly"/>
```

••

### cs226\_checks.xml (4)

```
<module name="ArrayTrailingComma"/>
       <module name="CovariantEquals"/>
                                               <!-- avoid accidental overload -->
       <module name="DeclarationOrder"/>
                                               <!-- standardize classes -->
       <module name="DefaultComesLast"/>
                                               <!-- standardize switch -->
       <module name="EmptyStatement"/>
       <module name="EqualsAvoidNull"/>
       <module name="EqualsHashCode"/>
       <module name="ExplicitInitialization"/> <!-- avoid initializing twice -->
       <module name="FallThrough"/> <!-- avoid forgetting breaks -->
       <module name="HiddenField">
                                             <!-- softened for constructors -->
          <property name="ignoreConstructorParameter" value="true"/>
       </module>
       <module name="IllegalCatch"/> <!-- avoid overly generic catch -->
       <module name="IllegalThrows"/> <!-- avoid overly generic throw -->
       <module name="InnerAssignment"/> <!-- avoid assignments as expressions -->
       <!--<module name="MagicNumber"/>--> <!-- more trouble than it's worth, we
can still grade them down but we don't have to force ridiculous constant
declarations -->
       <module name="MissingSwitchDefault"/> <!-- standardize switch -->
       <module name="ModifiedControlVariable"/>
       <module name="MultipleVariableDeclarations"/>
       <module name="NestedTryDepth"/> <!-- no try inside a try -->
       <module name="NoClone"/>
       <module name="NoFinalizer"/>
       <module name="OneStatementPerLine"/>
       <module name="OverloadMethodsDeclarationOrder"/>
       <module name="RequireThis"/>
                                              <!-- emphasize non-local stuff -->
       <module name="SimplifyBooleanExpression"/>
       <module name="SimplifyBooleanReturn"/>
       <module name="StringLiteralEquality"/> <!-- reminder to use equals() -->
```

L,

### cs226\_checks.xml (5)

```
<!-- annotation checks -->
<module name="AnnotationLocation"/>
```

```
<!-- standardize classes -->
```

```
<module name="NPathComplexity"/>
```

```
<!-- miscellaneous checks -->
<module name="ArrayTypeStyle"/>
<module name="CommentsIndentation"/>
<module name="Indentation"/>
<module name="OuterTypeFilename"/>
<module name="TodoComment"/>
<module name="UpperEll"/>
```

\$ java -jar datastructures2018/resources/checkstyle-8.12-all.jar \
 -c datastructures2018/resources/cs226\_checks.xml HelloWorld.java

### checkstyle.sourceforge.org

• • • iscellaneous × +					
$\leftarrow \rightarrow \mathbb{C} \ \triangle  \textcircled{O} \ \text{Not Secure} \   \ \textbf{checkstyle.sourceforge.net/config_misc.html#UpperEll} \qquad \Rightarrow  \textcircled{O} \ \notin \ M: \ \bigcirc \ \fbox{O} \ \textcircled{O} \ \textcircled{O} \ \textcircled{O} \ \textcircled{O} \ \swarrow \ \textcircled{O} \ \end{array} \end{array} \end{array}$ {O} \e					
🛂 🔯 💁 JHUMail 🛅 Daily 🛐 🛐 🎐 🏈 🗅 SL 🔿 🔯 🝐 📶 🐖 🗢 🛅 cshl 🚞 jhu 🛅 Media 🛅 shop 🗅 edit 🗅 Rm Cookies 🗅 Remove NYT Coo 🛛 » 🗎 Other Bookmarks					
UpperEll					
Description					
Since Checkstyle 3.0					
Checks that long constants are defined with an upper ell. That is ' L' and not '1'. This is in accordance with the Java Language Specification, Section 3.10.1 🖉.					
The lower-case ell '1' looks a lot like 1.					
Examples					
To configure the check:					
<module name="UpperEll"></module>					
Example Of Usage					
<ul> <li>Google Style ♥</li> <li>Sun Style ♥</li> <li>Checkstyle Style ♥</li> </ul>					
Error Messages					
• upperEll 🤣					
All messages can be customized if the default message doesn't suit you. Please see the documentation to learn how to.					

Package

# Agenda

- I. Review HWI
- 2. Introduce HW 2
- 3. Recap & continuation on complexity

# **Complexity Analysis**

#### How long will the algorithm take when run on inputs of different sizes:

• If it takes X seconds to process 1000 items, how long will it take to process twice as many (2000 items) or ten times as many (10,000 items)?

Generally looking for an order of magnitude estimate:



#### Also very important for space characterization:

Sometimes doubling the number of elements will more than double the amount of space needed

# Find Max: Linear Search (1)

import java.text.NumberFormat;

```
public class ArrayFind {
  final static int MAXINT = 100000000;
  // return the biggest int in the array
  public static int findMaximum(int [] myarray){
  }
  public static void main(String[] args) {
    if (args.length == 0) {
      System.out.println("USAGE: ArrayFind <array size>");
      return;
    }
    int arraysize = Integer.parseInt(args[0]);
    System.out.println("Scanning the array of size: " +
                         NumberFormat.getInstance().format(arraysize));
    int [] myarray = new int[arraysize];
```

# Find Max: Linear Search (2)

. . .

}

```
int [] myarray = new int[arraysize];
  // initialize with random valuas
  for (int i = 0; i < myarray.length; i++) {</pre>
    int random = (int)(Math.random() * MAXINT);
    myarray[i] = random;
  }
  long startTime = System.nanoTime();
  int max = findMaximum(myarray);
  long endTime = System.nanoTime();
  long duration = endTime - startTime;
  System.out.println("The max is: " + max);
  System.out.println("Search took: " +
      NumberFormat.getInstance().format(duration) + " nanoseconds");
}
```

## FindMax Analysis

```
public static int findMaximum(int [] myarray) {
    int max = myarray[0];
    for (int i = 1; i < myarray.length; i++) {
        if (myarray[i] > max) {
            max = myarray[i];
        }
    }
    return max;
}
```

\$ java ArrayFind 10000000 Scanning the array of size: 10,000,000 The max is: 99999989 Search took: 11,666,963 nanoseconds

\$ java ArrayFind 10000000 Scanning the array of size: 100,000,000 The max is: 99999999 Search took: 71,270,945 nanoseconds

Why isnt ArrayFind 100M exactly 10 times longer than ArrayFind 10M?

# FindMax Analysis

```
public static int findMaximum(int [] myarray) {
    int max = myarray[0];
    for (int i = 1; i < myarray.length; i++) {
        if (myarray[i] > max) {
            max = myarray[i];
        }
    }
    return max;
```

}

#### How many comparisons are done?

i < myarray.length n myarray[i] > max n C(n) = 2n

How many assignments are done (worst case)?

max = myarray[0]	I
<pre>for i =1; i &lt; myarray.length; i++</pre>	n
val = myarray[i]	n-l
max = myarray[i]	n-l
A(n) = I+ n + 2(n-I) = 3n-I	

# FindMax Analysis

```
public static int findMaximum(int [] myarray) {
    int max = myarray[0];
    for (int i = 1; i < myarray.length; i++) {
        if (myarray[i] > max) {
            max = myarray[i];
        }
    }
    return max;
```

}

What is the total amount of work done?

$$T(n) = C(n) + A(n) = (2n) + (3n - 1) = 5n - 1$$

Should we worry about the "-1"?

Nah, for sufficiently large inputs will make a tiny difference

Should we worry about the 5n?

Nah, the runtime is linearly proportional to the length of the array

# **Big-O** Notation

- Formally, algorithms that run in O(X) time means that the total number of steps (comparisons and assignments) is a polynomial whose largest term is X, aka asymptotic behavior
  - $f(x) \in O(g(x))$  if there exists c > 0 (e.g., c = 1) and  $x_0$  (e.g.,  $x_0 = 5$ ) such that  $f(x) \le cg(x)$  whenever  $x \ge x_0$ 
    - T(n) = 33 => O(1)
    - T(n) = 5n-2 => O(n)
    - $T(n) = 37n^2 + 16n 8$
    - $T(n) = 99n^3 + 12n^2 + 70000n + 2$
    - $T(n) = 127n \log (n) + \log(n) + 16$
    - $T(n) = 33 \log(n) + 8$
    - $T(n) = 900 \times 2^n + 12n^2 + 33n + 54$
- $\begin{array}{ll} 3 & => O(\lg n) \\ n^2 + 33n + 54 & => O(2^n) \end{array}$

 $=> O(n^2)$ 

 $=> O(n^3)$ 

 $=> O(n \lg n)$ 

- Informally, you can read Big-O(X) as "On the order of X"
  - O(I) => On the order of constant time
  - O(n) => On the order of linear time
  - $O(n^2) => On$  the order of quadratic time
  - $O(n^3) => On$  the order of cubic time
  - $O(\lg n) => On$  the order of logarithmic time
  - $O(n \lg n) => On$  the order of  $n \log n$  time



A quadratic function isnt necessarily larger than a linear function for all possible inputs, but eventually will be

That largest polynomial term defines the Big-O complexity



A quadratic function isnt necessarily larger than a linear function for all possible inputs, but eventually will be

That largest polynomial term defines the Big-O complexity



A quadratic function isnt necessarily larger than a linear function for all possible inputs, but eventually will be

That largest polynomial term defines the Big-O complexity

# Pop quiz!

```
public static int foo(int n) {
    int sum = 0;
    for (int i = 0; i < n; i++) {
        sum = sum + i;
    }
    return sum;
}</pre>
```

```
What is foo(10)?
```

What is the complexity?

```
public static int bar(int n) {
    int sum = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            for (int k = 0; k < j; k++) {
                sum = sum + i + k + k;
            }
        }
    return sum;
}</pre>
```

What is bar(10)?

What is the complexity?

# Pop quiz!

```
public static int baz(int n) {
    if (n <= 0) {
        return 0;
    }
    int sum = 0;
    for (int i = 0; i < n; i++) {
        sum = sum + i;
    }
    return sum + baz(n/2) + baz(n/2-1);</pre>
```

What is baz(10)?

What is the complexity?

```
public static int buz(int n) {
    if (n <= 1) {
        return 1;
    }
    return buz(n-1) + buz(n-2);
}</pre>
```

}

What is buz(10)?

What is the complexity?

# Fibonacci Sequence



# Fibonacci Sequence



[What is the running time?]

# Bottom-up Fibonacci Sequence

```
public static int fastbuz(int n) {
    int [] s = new int[n+1];
    s[0] = 1; s[1] = 1;
    for (int i = 2; i <= n; i++) {
        s[i] = s[i-1] + s[i-2];
    }
    return s[n];
}</pre>
```



[How long will it take for F(7)?] [What is the running time?]



# Fib vs FastFib

\$ for i in `seq 1 50`; do echo \$i; java Buz \$i; done 1 Scanning the array of size: 1 The value is: 1 Search took: 3,515 nanoseconds 2 Scanning the array of size: 2 The value is: 2 Search took: 3,849 nanoseconds 3 Scanning the array of size: 3 The value is: 3 Search took: 4,034 nanoseconds . . . 47 Scanning the array of size: 47 The value is: 512559680 Search took: 11,723,622,912 nanoseconds 48 Scanning the array of size: 48 The value is: -811192543 Search took: 19,283,637,425 nanoseconds 49 Scanning the array of size: 49 The value is: -298632863 Search took: 33,963,346,264 nanoseconds 50 Scanning the array of size: 50 The value is: -1109825406 Search took: 51,185,363,592 nanoseconds \$ for i in `seq 1 50`; do echo \$i; java FastBuz \$i; done 1 Scanning the array of size: 1 The value is: 1 Search took: 4,116 nanoseconds 2 Scanning the array of size: 2 The value is: 2 Search took: 4,286 nanoseconds 3 Scanning the array of size: 3 The value is: 3 Search took: 4,600 nanoseconds . . . 47 Scanning the array of size: 47 The value is: 512559680 Search took: 9,140 nanoseconds 48 Scanning the array of size: 48 The value is: -811192543 Search took: 10,143 nanoseconds 49 Scanning the array of size: 49 The value is: -298632863 Search took: 9,212 nanoseconds 50 Scanning the array of size: 50 The value is: -1109825406 Search took: 9,662 nanoseconds

# Dynamic Programming

- General approach for solving (some) complex problems
  - When applicable, the method takes far less time than naive methods.
    - Polynomial time (O(n) or O(n<sup>2</sup>) instead of exponential time (O(2<sup>n</sup>) or O(3<sup>n</sup>))
- Requirements:
  - Overlapping subproblems
  - Optimal substructure
- Applications:
  - Fibonacci
  - Longest Increasing Subsequence
  - Sequence alignment, Dynamic Time Warp, Viterbi
- Not applicable:
  - Traveling salesman problem, Clique finding, Subgraph isomorphism, ...
  - The cheapest flight from airport A to airport B involves a single connection through airport C, but the cheapest flight from airport A to airport C involves a connection through some other airport D.



# Dynamic Programming

- General approach for solving (some) complex problems
  - When applicable, the method takes far less time than naive methods.
    - Polynomial time (O(n) or O(n<sup>2</sup>) instead of exponential time (O(2<sup>n</sup>) or O(3<sup>n</sup>))
- Requirements:
  - Overlapping subproblems
  - Optimal substructure
- Applications:
  - Fibonacci
  - Longest Increasing Subsequence
  - Sequence alignment, Dynamic Time Warp, Viterbi
- Not applicable:





Horrible Bad Fair Good Excellent





# Data Structure Complexities

#### **Common Data Structure Operations**

Data Structure	Time Complexity						Space Complexity		
	Average				Worst			Worst	
	Access	Search	Insertion	Deletion	Access	Search	Insertion	Deletion	
Array	θ(1)	<mark>θ(n)</mark>	<mark>θ(n)</mark>	<mark>θ(n)</mark>	0(1)	0(n)	0(n)	0(n)	0(n)
Stack	θ(n)	<mark>Θ(n)</mark>	0(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Queue	θ(n)	<mark>Θ(n)</mark>	0(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Singly-Linked List	θ(n)	<mark>θ(n)</mark>	θ(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Doubly-Linked List	θ(n)	<mark>Θ(n)</mark>	Θ(1)	0(1)	0(n)	0(n)	0(1)	0(1)	0(n)
Skip List	θ(log(n))	O(log(n))	θ(log(n))	0(log(n))	0(n)	0(n)	0(n)	0(n)	0(n log(n))
Hash Table	N/A	0(1)	θ(1)	0(1)	N/A	0(n)	0(n)	0(n)	0(n)
Binary Search Tree	θ(log(n))	θ(log(n))	θ(log(n))	θ(log(n))	0(n)	0(n)	0(n)	0(n)	0(n)
Cartesian Tree	N/A	O(log(n))	O(log(n))	0(log(n))	N/A	0(n)	0(n)	0(n)	0(n)
B-Tree	θ(log(n))	θ(log(n))	θ(log(n))	θ(log(n))	0(log(n))	0(log(n))	0(log(n))	0(log(n))	0(n)
Red-Black Tree	θ(log(n))	θ(log(n))	θ(log(n))	θ(log(n))	0(log(n))	0(log(n))	0(log(n))	0(log(n))	0(n)
Splay Tree	N/A	θ(log(n))	θ(log(n))	θ(log(n))	N/A	0(log(n))	0(log(n))	0(log(n))	0(n)
AVL Tree	θ(log(n))	θ(log(n))	θ(log(n))	θ(log(n))	0(log(n))	0(log(n))	0(log(n))	0(log(n))	0(n)
KD Tree	θ(log(n))	θ(log(n))	θ(log(n))	θ(log(n))	<mark>0(n)</mark>	0(n)	0(n)	0(n)	0(n)

#### http://bigocheatsheet.com/

# **Next Steps**

- I. Submit HWI
- 2. Work on HW2
- 3. Check on Piazza for tips & corrections!